The hipster effect: When anticonformists all look the same

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In such different domains as statistical physics and spin glasses, neurosciences, social science, economics and finance, large ensemble of interacting individuals taking their decisions either in accordance (mainstream) or against (hipsters) the majority are ubiquitous. Yet, trying hard to be different often ends up in hipsters consistently taking the same decisions, in other words all looking alike. We resolve this apparent paradox studying a canonical model of statistical physics, enriched by incorporating the delays necessary for information to be communicated. We show a generic phase transition in the system: when hipsters are too slow in detecting the trends, they will keep making the same choices and therefore remain correlated as time goes by, while their trend evolves in time as a periodic function. This is true as long as the majority of the population is made of hipsters. Otherwise, hipsters will be, again, largely aligned, towards a constant direction which is imposed by the mainstream choices. Beyond the choice of the best suit to wear this winter, this study may have important implications in understanding dynamics of inhibitory networks of the brain or investment strategies finance, or the understanding of emergent dynamics in social science, domains in which delays of communication and the geometry of the systems are prominent.

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Hipsters avoid labels and being labeled. However, they all dress the same and act the same and conform in their non-conformity. Doesn't the fact that there is a hipster look go against all hipster beliefs? This perspicacious observation of the blogger Julia Plevin [1] in 2008 proves true along the years, and 2014 hipsters all look alike, although their look progressively evolves. The *hipster effect* is this non-concerted emergent collective phenomenon of looking alike trying to look different. Uncovering the structures behind this apparent paradox goes beyond finding the best suit to wear this winter. They can have implications in deciphering collective phenomena in economics and finance, where individuals may find an interest in taking positions in opposition to the majority (for instance, selling stocks when others want to buy). Applications also extend to the case of neuronal networks with inhibition, where neurons tend to fire when others and silent, and reciprocally.

The question, as well as methods developed in the domain, are evocative of a wide literature dealing with large systems of interacting agents making random choices with probabilities depending on the choice of the majority. Models were developed in such different domains as the alignment of spins in magnets [2, 3], transmission of electrical information in networks of neurons [4, 5], and choices in economics and social science [6]. In a general framework, we consider here populations made of *hipsters*, or anticonformists, who take their decisions in opposition to the majority, and *mainstream* individuals that tend to follow the majority. The main novelty is to take into account the time needed by each individual to feel the trend of the majority. These delays, related to the dynamics of the interactions and sometimes the geometry of the system, are often neglected in physics or social systems, but it is well known to computational neuroscientists that the time taken by the conduction of neuronal influx along the axons shape the collective dynamics of coupled cells [7, 8]. In economics as well as in social science and opinion dynamics, the time taken by the information to be transmitted to the network, as well as the relative influence of the past on the decisions taken seem prominent: a given individual needs some time to receive and take into account a decision of an other individual. Moreover, specific individuals are more influential than others, at least to the eyes of some. This heterogeneity in the way trends are perceived are central in applications. However, our understanding of their impact on collective dynamics is still poor, and is essential to understand the dynamics as we show here. We will concentrate on a simple canonical model and show that depending on delay and heterogeneity distribution, qualitative dynamics are substantially modified, and these can even lead to a dramatic synchronization of hipsters' choices.

We will investigate these questions in a generic model of spin glasses. We consider n individuals, that randomly switch between two states $\{-1, 1\}$, depending on their hipster or mainstream nature and the majority trend they feel. Individual i is randomly chosen to be hipster with probability q, or mainstream, and therefore the type of individuals in the population is characterized by the sequence of random variables $(\varepsilon_i)_{i=1\cdots n} \in \{-1, 1\}^n$, drawn prior to the evolution of the network and frozen during time evolution (we choose by convention $\varepsilon = -1$ for hipsters). Each individual is defined by its current state $s_i(t)$ and the network state is described by a vector

$\mathbf{s}(t) \in \{-1, 1\}^n$, that switches randomly with a probability varying with the mean-field trend $m_i(t)$ felt by i, which depends on (i) how the individual i sees its environment and (ii) on the history of the system. In detail, individual i assigns a fixed weight $J_{ij} \geq 0$ to individual j, and only sees its state after a delay τ_{ij}, so that the trend seen by individual i at time t is simply:

$$m_i(t) = \frac{1}{n} \sum_j J_{ij} s_j(t - \tau_{ij})$$

We will make the assumption that for any fixed i, the weights and delays pairs $(J_{ij}, \tau_{ij})_{j=1\cdots n}$ are independent and identically distributed random variables (or with law $p_{\varepsilon_i,\varepsilon_j}$ that only depend on the type of i and j), drawn prior to the stochastic evolution of the network, and frozen along the evolution in time. They constitute a random environment. From the modeling viewpoint, the weight J_{ij} is large if j has a prominent impact on the choice of i, and it is null if decisions of j do not impact i. An important example that we treat here is when both delays and weights depend on a hidden variable, which is the relative location in space of the two individuals.

Once a configuration is fixed, individuals evolve according to a random Markov process. Given the state $\mathbf{s}(t)$ of the network at time t, each individual makes the switch $s_i \to -s_i$ as an inhomogeneous Poisson process with rate $\varphi(-\varepsilon_i\, m_i(t)\, s_i)$ where φ is a non-decreasing sigmoid function centered at zero. In that model, if the state $s_i(t)$ is opposite to the felt trend $m_i(t)$, mainstream individual ($\varepsilon_i = 1$) have a higher switching rate, and hipsters a lower switching rate. The gain of the sigmoid $\varphi(\cdot)$ is directly related to the level of noise. To fix ideas, we chose $\varphi(x) = 1 + \tanh(\beta x)$, where $\beta > 0$ is called inverse temperature, and governs the sharpness of the rate function: the larger β, the sharper $\varphi(x)$ and therefore the less random the transition.

Before developing our theory, let us spend some time describing the relationship between this model and more classical spin-glass systems. Beyond the presence of delays that are specific to the present model, we consider asymmetric interactions, meaning that the action of individual i on j is of the same amplitude as the reciprocal action of j on i. In that sense, our system is comparable to binary neuron models as introduced in early works in the domain [9]. Another difference appears in the way we incorporate the mainstream-hipster nature as a characteristic of each individual, which differs from works done in neuroscience or in the Sherrington-Kirkpatrick spin glass system [2, 10] in which interaction between i and j is generally assumed to have a random sign, which is independent pair are positive to occur with positive or negative amplitude depending on the pair (i, j) considered.

This model however remains simple enough to be completely solvable: one can find a closed-form solution for the thermodynamic limit of the system, in terms of a self-consistent jump process with rate depending on the statistics of the solution. Moreover, the average behavior is exactly reduced to a set of delayed differential equations, and therefore we will be able to use the bifurcation theory developed in this context to uncover phase transitions related to the delays distribution. This is how we will be able to show rigorously how delays induce a synchronization of hipsters.

The thermodynamic limit of the system can be described as a jump process whose jump statistics depend on a self-consistent quantity. In detail, in the limit $n \to \infty$, individuals behave independently (a property similar to Boltzmann's molecular chaos, called propagation of chaos property in mathematics [11]), and therefore the jump rate $s \to -s$ averages out to $\varphi(\varepsilon \rho_\varepsilon(t) s)$ with

$$\rho_\varepsilon(t) = \sum_{\varepsilon'=\pm 1} q_{\varepsilon'} \int_{\mathbb{R}^2} j m_{\varepsilon'}(t - \tau u) dp_{\varepsilon,\varepsilon'}(j, \tau)$$

where $m_\varepsilon(t) := \mathbb{E}[s_\varepsilon(t)]$ is the averaged value (statistical expectation) of individuals of type ε at time t and $q_\pm$ the proportion of conformists (q) and anticonformists $(1-q)$ individuals. Heuristically, each individual's jump intensity is the rate of one process, averaged statistically and also averaged over all possible configurations of weights J_{ij}, delays τ_{ij} and individual types. A rigorous mathematical proof can be done using the theory of McKean-Vlasov limit theorems for jump processes developed in the 1990s [12–14]. Specific care has to be taken in our case, since we deal with (i) delayed systems that require to use infinite-dimensional state spaces of trajectories $(\mathbf{s}(u))_{u\in[t-\tau,0]}$, and (ii) random environments, but the principle of the proof however remains identical.

The mean-field equation is a priori complex: it is a non-Markov process in the sense that the jump rate of a given solution depends on the law of the solution and not on the value of the process itself. However, thanks to the simplicity of the model, we can characterize very precisely the probability distribution of this process. Indeed, the thermodynamic limit is univocally described by the two jump rates $\rho_\varepsilon(t)$, that only depend on the knowledge of the average state of individuals in the two populations m_ε. It is not hard to show that the latter variables are solution to the differential equation with distributed delays:

$$\dot{m}_\varepsilon(t) = -2\Big(m_\varepsilon(t) + \tanh(-\varepsilon\beta\rho_\varepsilon(t))\Big).$$

These equations allow to analyze rigorously the system and the role of different parameters. We concentrate on two simple situations in which the role of the different parameters are disentangled: (A) a case with constant communication and delay coefficients independent of the individual type, and (B) a case where delays and communication coefficients are both dependent on a hidden random parameter, the respective locations of the different individuals, treated for simplicity in a pure hipster situation.

In order to characterize the phase transitions in the system, we investigate the linear stability of the disordered solution $m_\varepsilon = 0$, which depends on the spectrum of the linearized operator given by the solutions of the dispersion relationship

$$\dot{\zeta}_\varepsilon = -2\zeta_\varepsilon - 2\varepsilon\beta \sum_{\varepsilon'=\pm 1} q_{\varepsilon'} \int_{\mathbb{R}^2} j e^{-\tau\zeta_{\varepsilon'}} dp_{\varepsilon,\varepsilon'}(j,\tau). \qquad (1)$$

Let us start by dealing with situation (A) where $p_{\varepsilon,\varepsilon'} = \delta_{\bar{J},\tau}$. The variable $z = qm_{+1} + (1-q)m_{-1}$, the total trend over the whole population, satisfies the equation:

$$\dot{z} = -2\Big(z + (2q-1)\tanh(\beta \bar{J} z(t-\tau))\Big)$$

and from now on consider without loss of generality $\bar{J} = 1$. The linearized equations around the disordered equilibrium ($z = 0$) greatly simplify, and it is then easy to characterize stability by finding the characteristic roots of the system:

$$\lambda = -2(1 + (2q-1)\beta e^{-\lambda\tau})$$

For $\tau = 0$, 0 is stable for $1 + (2q-1)\beta > 0$ and unstable otherwise. This implies that populations in which anticonformists are majoritary ($q > 1/2$) never find consensus, while populations dominated by conformists can find a consensus, but at sufficiently small temperature. In detail, consensus are found for β larger than a critical value $\beta_c(q)$ that increases with the proportion of hipsters

$$\beta_c(q) = \frac{1}{1-2q}.$$

Below the noise level, the disordered state $z = 0$ looses stability and a state with non-zero trend is found. Heuristically, as long as anticonformists are majoritary, they will compensate instantaneously any alignment of the mainsteam individuals, and therefore prevent any magnetization to emerge. But when there is a majority of mainstream individuals, a trend may emerge if the level of randomness in their choices is small enough. Hipsters will then consistently oppose to this trend, creating a clear non-trivial hipster trend. The fact that the level of noise at which this equilibrium emerges is lower than that of a pure ferromagnetic spin glass system can be interpreted as the fact that, from a microscopic viewpoint, the systematic furstration and misalignment of hipsters results in an increased effective temperature. Precisely at the critical transition $q = 1/2$, very complex phenomena appear, where populations of anticonformists and hipsters align transiently, in a non-periodic manner, before switching at random times. A typical example is plotted in Fig. 1 (a).

Instead, we shall concentrate of the role of the delays. For $\beta < \beta_c(q)$, it is easy to see that 0 is the unique stable solution. Indeed, shall there exist such (β, τ) for which 0 is unstable, we would then have characteristic roots $\lambda = a + \mathbf{i}b$ with positive real parts, i.e. such that:

$$a = -2 + 2(1-2q)\beta e^{-a\tau}\cos(b\tau)$$

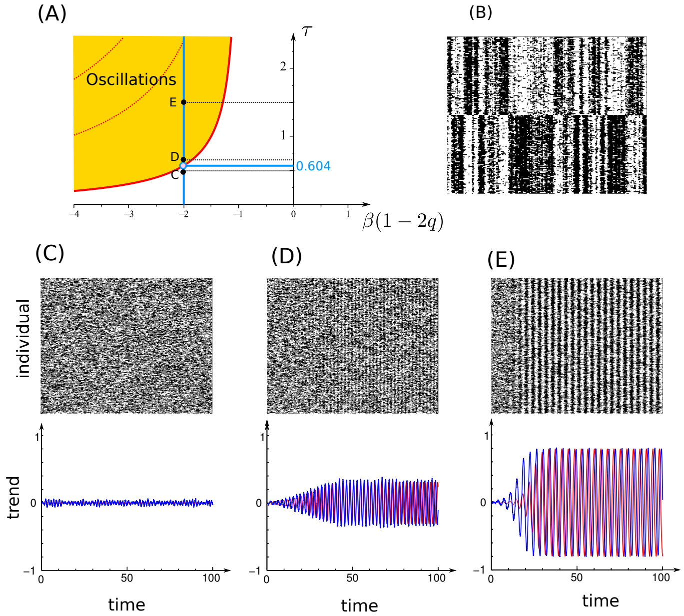

FIG. 1. (A) Delay-induce Hopf bifurcation in the plane $(\hat{\beta}, \tau)$ with $\hat{\beta} = \beta(1-2q)$. (B-E) simulations of the discrete system for $n = 5\,000$, $\beta = 2$. (B) $q = \frac{1}{2}$: phase transition. (C-E): $q = 1$ (fully anti-conformist system) and different delays $\tau = 0.5$ (C), 0.7 (D) and 1.5 (E) respectively. Top row: time evolution of all particles as a function of time, bottom row: empirical (blue) and theoretical (red) total trend.

but $|2(2q-1)\beta e^{-a\tau}\cos(b\tau)| < 2$ hence this is impossible.

However, at low temperature ($\beta > \beta_c(q)$) a destabilization may occur. Shall this happen, the characteristic roots will cross the imaginary axis, and this is only possible for purely imaginary eigenvalues of the linearized operator. Algebraic manipulations yield to the fact that Hopf bifurcations occur along the following curve in the parameter space:

$$\tau = \frac{\pi + \arctan(\sqrt{(2q-1)^2\beta^2 - 1}) + 2k\pi}{2\sqrt{(2q-1)^2\beta^2 - 1}} \qquad k \in \mathbb{Z}.$$

A non-trivial solution therefore emerges, which oscillates between positive and negative values. The individuals remain synchronized, even if their orientation is not stationary, but switches very regularly, in a periodic manner, between positive and negative.

Heuristically, this oscillatory phenomenon arises from the slowness of the information transmission. Indeed, during the evolution of the network, fluctuations of the trend will tend to be amplified by the delay mechanism. Indeed, a random imbalance will be detected after some time and all anticonformist individuals will tend to disalign to this trend, regardless of the fact that an increasing proportion of them do and therefore yield a clear bias towards the opposite trend. This will be detected at later times, leading to a reciprocal switch, and these oscillations will periodically repeat. Despite their efforts, at all times, anticonformists fail being disaligned with the majority.

To conclude with, we shall concentrate on a more realistic situation (B). We now consider that the environment variables (J_{ij}, τ_{ij}) have a pure geometric dependence: these are deterministic functions of the dissimilarity between the individual i and j, for instance a physical or functional distance between them. In that setting, we randomness depends on a hidden variable r_i, which is assumed for instance to take values on a compact set chosen to be the one-dimensional circle of length a, $\mathbb{S}^1_a$, and assume that this correspond to the location of individuals in a physical space. In that setting, individuals communicate after a time proportional to the distance between then added to a constant delay τ_0 corresponding to the transmission of information $\tau_{ij} = \tau_0 + |r_{ij}| =: T(r_{ij})$ with r_{ij} is the distance between i and j (on the circle). The distribution of the distance can be computed in closed-form: it has linearly decaying slope $d\eta(r) = \left(\frac{2}{a} - \frac{2r}{a^2}\right) dr$. Coefficients J_{ij} take into account the fact that distant individuals have a smaller probability to communicate. The probability that two individuals at a distance r communicate with each other is assumed to decays with a profile $\psi(r)$, and the communication strength is assumed constant equal $\bar{J} > 0$ [15]. In other words, $J_{ij} = \bar{J}\xi_{ij}$ with ξ_{ij} a Bernoulli random variable of parameter $\psi(r_{ij})$. from which we find the probabilities of the pairs (J_{ij}, τ_{ij}) given by the density $dp(j,\tau) = \int_{\mathbb{S}^1} \delta_{\{j=\psi(r), \tau=T(r)\}} \eta(r) dr$. This allows to compute the linearized operator, and find the Hopf bifurcation curve in the space of delays and size a. In detail, for $\psi(r) = e^{-\gamma r}$, the eigenvalues ξ of the linearized operators are solutions of the dispersion relationship:

$$\xi = -2\left(1 - \beta\bar{J}\int_0^a e^{-(\gamma+\xi)r}\left(\frac{2}{a} - \frac{2r}{a^2}\right) dr\right)$$

and therefore Hopf bifurcations arise only if one can find parameters of the model, and a positive quantity $\omega > 0$, satisfying the relationship:

$$\mathbf{i}\omega = -2 + \frac{2\beta\bar{J}}{a(\gamma+\mathbf{i}\omega)}\left(1 - \frac{1}{a(\gamma+\mathbf{i}\omega)} + \frac{e^{-a(\gamma+\mathbf{i}\omega)}}{a(\gamma+\mathbf{i}\omega)}\right) e^{-\mathbf{i}\omega\tau_s}. \tag{2}$$

This equation cannot be solved in closed form as in the previous case, but however it is easy to express the locus of the Hopf bifurcation in the parameters space (a, τ_0) as a parametric curve, and therefore access with arbitrary precision to the Hopf bifurcation in the plane (see Fig. 2).

This curve has a very interesting, non-monotonic shape. It shows that there is an optimal spatial extension of the hipster population most favorable for synchronization: populations spreading on too small or too large intervals will not synchronize, and there exists a specific length interval in which hipsters synchronize. This effect is actually the result of two competing mechanisms: increasing the size of the interval makes the average delay increase (as $a/2$), but the variance of the delays increases as well, which reduces the coherence of the signal received, and may make synchronization harder.

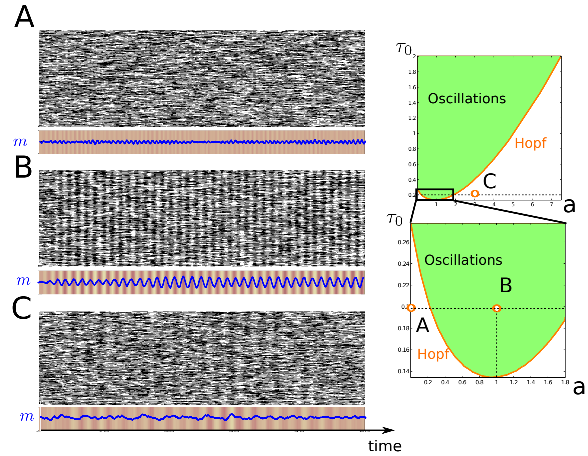

FIG. 2. Space-dependent delays and connectivity: bifurcations as a function of the length a of the interval on which hipsters communicate. Parameters $\beta p = 4$, $\gamma = 0.3$, $\tau_s = 0.2$, length of the interval: (A) $a = 0.1$ and (C): $a = 3$, no synchronization, (B): $a = 1$, synchronization. Simulation of the Markov chain with $N = 1000$ together with the computed trend below (computed averaged, plotted against a background with color proportional to the trend).

We therefore showed that, in contrast to cooperative systems, populations of individuals that take decision in opposition to the majority undergo phase transitions to oscillatory synchronized states if we take into account the delays in the communication between these individuals. This study opens the way to the understanding of synchronization and correlations in other statistical models, such as those developed in finance, in which case speculators may make profit when taking decisions in opposition to the majority in stock exchange. This problem has been the subject of intensive researches around the so-called minority games (see the book [16] presenting motivations and models), which our system is a particular case of. The analysis of the relatively simple model allowed to go very far in the understanding of the concurrent role of noise, delays and proportions of hipsters and mainstream individuals in this emergence of synchronization among hipsters. Interestingly, synchronization may depend on the precise shape of the distribution of the delays: for synchronization to emerge, one needs both sufficiently long delays and sufficient coherence (small standard deviation of the delays). This yielded the unexpected phenomenon that synchronization among hipsters depends on the distribution, in space, of each individuals, when the delays are function of the distance between two individuals. Along the way, we uncovered several points that are well worth studying in depth. For instance, the behavior of a system with an equal proportion of hipsters and mainstreams appears to be a singular phase transition in which the whole population tends to randomly switch between different trends, and would be very interesting to further characterize.

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