Fact, Fiction, and Value Investing

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Abstract

Value investing has been a part of the investment lexicon for at least the better part of a century. In particular the diversified systematic "value factor" or "value effect" has been studied extensively since at least the 1980s. Yet, there are still many areas of confusion about value investing. In this article we aim to clarify many of these matters, focusing in particular on the diversified systematic value strategy, but also exploring how this strategy relates to its more concentrated implementation. We highlight many points about value investing and attempt to prove or disprove each of them, referencing an extensive academic literature and performing simple tests based on easily accessible, industry-standard public data.

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Introduction

While recently confronting the myths surrounding momentum investing[1] we discovered two things: 1) there is as much confusion about value investing and 2) if one debunks the mythology around momentum investing some will get the wrong impression that defending momentum means denigrating value. Even experienced investors often seem to wrongly assume one cannot simultaneously believe in both value and momentum investing.

Value is the phenomenon that securities which appear "cheap" on average outperform securities which appear to be "expensive." The value premium is the return achieved by buying (being long in an absolute sense or overweight relative to a benchmark) cheap assets and selling (shorting or underweighting) expensive ones. The existence of the value premium is a well-established empirical fact: it is evident in 87 years of U.S. equity data, in over 30 years of out-of-sample evidence from the original studies, in 40 other countries, in more than a dozen other asset classes,[2] and even dating back to Victorian age England![3]

Importantly, our definition of "value investing" is the highly diversified "academic" (though many practitioners follow it also) version of value, not concentrated value-based stock picking (which we discuss further in one of our sections). In addition, our starting point is "pure" value, price vs. some fundamental like book value, based solely on quantifiable measures and not adjusted for other qualities (a distinction which we will also address in this paper). For example, by our definition, a "pure" value investor only looks to purchase low price-to-book stocks. "Adjusting for other qualities" means you are willing to pay more, accept a higher price-to-book, for faster growers, more profitable firms, or any other quality you desire (some call this "growth at a reasonable price" though obviously we are generalizing beyond just growth). The interplay between "pure value" and value considering other qualities will come up a few times in the paper.

Value strategies have had a long and storied history in financial markets. They are often credited to Benjamin Graham and David Dodd dating back to the late 1920s, who advocated a form of value investing by buying profitable but undervalued assets (a double condition we will soon see to be an important distinction from what we call "pure value"). Indeed, value investing has been considered an important part of the equity investment landscape for the better part of the last century, and while undocumented, we would assume for far longer (somewhere there must have been a Roman saying "I came, I saw, I purchased at a low multiple"). However, despite all of this, there still remains much confusion about value investing. Some of this confusion is propagated by value's opponents in an attempt to disparage the strategy, but others are often,

[1] "Fact, Fiction, and Momentum Investing," Journal of Portfolio Management [2014], 40th Anniversary edition.
[2] See Asness et al. [2013].
[3] See Chabot, Ghysels, and Jagannathan (2015), who show evidence of a value effect using dividend yield in U.K. stocks going back to the 1860's.

intended or not, also perpetuated by those explicitly or implicitly advocating it (there is even one notable case of a clear pure value investor who claims not to be!).

Our paper is organized by identifying a number of facts and fictions about value investing that need clarification. The facts we present include showing that value works best with other factors, which can still be consistent with risk-based explanations for value, that it is best measured by multiple variables (rather than a single variable such as book value relative to market value), that it is exactly what the recently popular investing approach called "Fundamental Indexing" does, and that it is a weak effect among large cap stocks (especially relative to other factors that hold up in both large and small caps). The fictions we will attempt to clarify include the false notions that value investing is only effective in concentrated portfolios, that it is a "passive" strategy, that it is a redundant factor in the face of newly emergent academic factors (namely, Fama and French's new investment and profitability factors), and that it is only applicable in a stock picking context. Finally, we will take on the commonly held belief that value is solely compensation for risk yet somehow not a scary strategy and that a continued value premium in the future can only be consistent with an efficient markets view of the world. We certainly do not reject efficient markets, but value's success can occur in a world of efficient markets, inefficient markets, or the likely truth that lies somewhere in-between, all of which are subject to time variation and hard to envision disappearing.

While our prior paper on momentum was an attempt to address and refute the myths and public criticisms hurled at momentum investing, this paper instead tries to present some facts and clarify some misconceptions surrounding value investing, a somewhat softer goal befitting a more "occasionally confused" than "attacked" strategy. As in the prior paper, we address the facts and fictions of value investing using published and peer-reviewed academic papers and conduct tests using the most well-known and straightforward publicly available data (most of which come from Kenneth French's website as detailed in the appendix) in U.S. equity markets.[4]

Finally, the topics we address will include both positive and negative attributes of value investing. We consider ourselves, and have for many years, among value investing's strongest proponents, particularly when used in combination with some other factors (like momentum and, more recently, profitability). Our discussion is not at all meant to denigrate a strategy that we believe is a corner-stone of good investing. Instead, it is merely to see the pros, cons, and even ancillary beliefs more clearly.

Now, on to the main event.

[4] Our results are extremely robust when ported to other countries and we invite others to verify this claim!

Fiction: Value investing can only be successfully implemented with a concentrated portfolio.

As we discussed in the introduction, in this paper we will explore the highly diversified, systematic version of "value investing", not concentrated value-based stock picking. Yet, to be a successful value investor don't you have to apply value in a concentrated portfolio, deeply understanding each and every security in order to uniquely identify cheap stocks? Warren Buffett certainly thinks so. To quote Mr. Buffett: "Diversification is protection against ignorance. It makes little sense if you know what you are doing."

As Mr. Buffett himself states, his common investment theme is to find "discrepancies between the value of a business and the price of that business in the market." He applies this philosophy to a handful of stocks that he deeply investigates and understands, holding them in a concentrated portfolio for the long-term. He's obviously done it incredibly well.

But, Benjamin Graham, who Buffett credits as a mentor for his investment training, actually believed in the long term evidence in favor of a *diversified* portfolio as opposed to one based on a few concentrated positions. In The Intelligent Investor (revised in 1973) he writes "In the investor's list of common stocks there are bound to be some that prove disappointing… But the diversified list itself, based on the above principles of selection, plus whatever other sensible criteria the investor may wish to apply, should perform well enough across the years. At least, long experience tells us so."

Still, does the existence of Warren Buffett and his long-term incredible performance prove that an idiosyncratic value process dominates a systematic one? As the saying at the University of Chicago goes, "the plural of anecdote is not data." Warren Buffett is to value investing what George Burns, the comedian who lived to 99 years of age while smoking 10 to 15 cigars a day for 70-plus years, is to the health effects of smoking.[5] Should we ignore the evidence that statistically you are at greater mortality risk from smoking based on George Burns? Unless you are the magazine *Cigar Aficionado*,[6] the answer is clearly, no. For value investing, there is strong long-term evidence from a wide variety of assets that a systematic value strategy can deliver good long-term returns. The fact that Warren Buffett was able to successfully pick individual cheap stocks should not derail that notion. Being Warren Buffett is nice work if you can get it, certainly after-the-fact. But, the legion of academic and practitioner evidence is that diversified portfolios of "cheap" (on pure value measures) securities healthily outperform their more

[5] We attribute a similar analogy to John Cochrane, who made it in a speech about Warren Buffett and market efficiency in honor of Eugene Fama's 2013 Nobel Prize in Economics. It also appears in print in Cochrane and Moskowitz [2014].

[6] *Cigar Aficionado* published an article in 1994 with the following incredulous quote: "Comedian George Burns is not only a living legend, he's living proof that smoking between 10 and 15 cigars a day for 70 years contributes to one's longevity." Or, as Mr. Burns put it himself, "If I'd taken my doctor's advice and quit smoking when he advised me to, I wouldn't have lived to go to his funeral." Of course, there is a good chance they were joking. We certainly hope so…

expensive brethren, all without the necessity (and the ex-ante danger) of picking the handful of best ones.

Of course, the systematic versus idiosyncratic concepts are not mutually exclusive. An investment professional that is very good at persistently identifying idiosyncratic cheap positions deserves a lot of credit. But a manager who is able to systematically invest in a group of cheap securities can also capture a positive long-term source of returns. An investor looking to invest in one of these two approaches should consider both as a way to diversify the process that generates the returns to value, as long as that investor is confident that the returns being generated by both processes are indeed persistent. In our view, the diversified, systematic process provides greater comfort in this regard, but that does not mean that a concentrated value process could not also add over time. The point is certainly not to denigrate Mr. Buffett's approach but to emphasize that value investing encompasses more than Warren Buffett's version. In fact, one can think of these two things separately. The returns of a diversified portfolio of value stocks over their more expensive counter-parts are available to any who choose to pursue them (but not if everyone chooses to pursue them, a different topic), and should be available at a reasonably low fee. Picking the exact right small handful of value stocks may, or may not, be possible, but comes with additional dangers (if you're wrong about one of a handful it matters a lot!) and additional upside (if you get it massively right for 50 years you get to be Warren Buffett!), and usually comes with a higher fee if purchased in the active management world (with Mr. Buffett being a pleasant exception here).

As an investor you may bet on one, both, or neither of these approaches subject to your own beliefs. We are here to mostly extoll (but also critically examine) the diversified value investing process, and to point out that it exists largely separate from the ability (or lack thereof) to build highly concentrated portfolios, without dismissing the latter possibility. At the very least we hope to convince the reader that these two investment philosophies are not mutually exclusive or indeed even in competition with one another.

Fiction: Value is a passive strategy because it is rules-based and has low turnover.

Although we differentiate between systematic value investing and the concentrated active stock picking of Warren Buffett, we often hear people go the other extreme, claiming that what we call systematic value is a passive strategy. In particular, some claim that value is passive, much like simply buying and holding the equity market index. The implication being that a value strategy does not make "active" choices and is therefore not "active management," which some value providers paint as dirty words in the investment world.

Our own view is that *anything* that deviates from the market portfolio, which weights assets in proportion to their market values, is by definition "active." The reason is the market portfolio is the only portfolio everyone can hold simultaneously (which is what makes it the market portfolio!). A portfolio that deviates from market weights, on the other hand, must be balanced

by other investors who are willing to take the other side of those bets. For example, for every value investor, who tilts toward or selects cheap value stocks, there has to be an investor on the other side who is underweight value and overweight expensive, growth stocks. Since everything has to add up to market weights, for every value investor there has to be a growth investor taking the other side. Hence, unlike the market portfolio, everyone at once cannot hold or tilt toward value. It is not a buy and hold strategy that everyone can do. Only the total market portfolio meets those criteria. In our view, this makes value an "active" choice, as it has to come from someone willing to take the other side (albeit we mean "active" here in a very different way than traditional, concentrated stock picking!) Any deviation from market weights is active by this definition – this is true of other systematic strategies such as momentum, profitability, low volatility, and, indeed again, value. We would argue the distinction people are after here is "judgmental" vs. "systematic", but instead they wrongly frame it as "active" vs. "passive".

Some might argue that our definition of passive is too narrow (and by extension our definition of active too broad) and that a better definition is the one alluded to by this statement: a passive strategy is one that follows simple rules and has low turnover. However, simple counter-examples show that this is not an appropriate definition of passive. Take, for example, a single stock portfolio that buys and holds that stock in perpetuity. Imagine an employee putting all of her wealth in the company she works for. By the above definition this would be considered a passive portfolio, even though it is clearly a concentrated, idiosyncratic, and active bet in one firm (that also happens to be tied to her human capital in our example). In fact it's massively active! Warren Buffett's portfolio would also be considered passive under this definition, too, since his turnover is even lower than the typical diversified, systematic value strategy. When single stock portfolios and Warren Buffett are falling under the definition of "passive", the definition seems flawed.

As for rules-based, this part of the definition fails, too. A high frequency trader who trades in microseconds by definition is rules-based, since humans are incapable of acting at such speeds, but we would be hard-pressed to call them passive investors.

Quite frankly, we find the debate between active and passive management not that useful. The main issues facing investors are really what they are getting (i.e., long-term expected returns with correlation properties that are valuable to their portfolio), at what price/cost, and are there good reasons, either risk or behavioral, to believe these returns will persist. Whether this comes from active or passive products is irrelevant so long as it adds value to their portfolio at the best price (there are good and bad active products, and good and bad passive ones, regardless of definition).

In our view, this is all semantics. Calling value "active" does not mean that it is bad for investors. On the contrary, we firmly believe that a systematic, disciplined, rules-based, and low turnover portfolio exposed to value (and other factors, too) is a great investment – no matter what you call it – and we would call it active.

Fact: "Fundamental Indexing" is, and only is, systematic value investing.

While some acclaimed value investors, such as Warren Buffett, are actually not pure value investors (they consider other "quality" measures too, something we discuss next), we now address the flip side of this issue – investors claiming *not* to be pure value investors, but who actually are.

There are a series of investment products with names like "Fundamental Indexing" (FI) and others that fall under the name "smart beta" that are just simple systematic tilts away from an index in the precise direction of value investing. That is fine. It is even great. In case it is not obvious, we are fans of systematic value investing! What is not fine is when people claim that their simple value tilts are something more, different or, even worse, a new discovery.

The violations of this in the case of Fundamental Indexing have been particularly acute. It started out more than a decade ago with early claims that Fundamental Indexing had near nothing to do with value investing. That was beyond the pale. It has evolved toward, but nowhere near all the way to, something more reasonable with claims that it is "related to" value investing but still somehow different and better.[7] Mixed in with this tends to be all kinds of theorizing about why value (or Fundamental Indexing) works. Much of that is also reinvented as if new (e.g., academics considered "noise" or mispricing as a potential reason for value's long-term success long before these musings) and serves to further obfuscate things, looking for points of distinction from value investing, where none exist. (By the way, we will have more to say on the theory behind why value works later.)

Fundamental Indexing works by weighting stocks according to various fundamentals (natch). These are things like book value, dividends, cash flows, sales, earnings, etc., as opposed to market capitalization, as done in traditional index funds. Proponents point out, correctly, that if prices contain errors (which of course they do, even efficient marketers don't believe in perfect efficiency) then a market capitalization based index by definition overweights the too expensive and underweights the too cheap (it is already sounding like we are discussing value investing, no?). Of course, prices can also vary for rational risk-based reasons, but we will come back to that issue later. In any case, weighting by fundamentals creates an investment product that does not contain this potential bias (though by no means do they provably take the optimal amount of value tilt – it might be too much or too little – just saying prices are not perfect does not say how much of a value bet to take). Also, unlike some popular but patently ridiculous (in terms of practicality) alternatives, like equal-weight indices, they create an investment product that is deep, liquid, and investable. All is good so far.

[7] The evolution away from initial claims of Fundamental Indexing being very different from value began even back in 2005. For example, Arnott, Hsu, and Moore [2005] say "A Fama-French three-factor regression shows that the Fundamental Indexes have factor exposure to the value factor." Of course, we would argue "have factor exposure" is a giant understatement!

But when its proponents overplay how this is more than value investing it adds confusion and hides the truth.

An equation illustrates the point nicely. If you create a Fundamental Index based on one measure (e.g., book value), the weight of stock *i* held in the Fundamental Index is the following function of its weight in a traditional market capitalization weighted index (assuming it is done over the same universe – varying the universes is another way to obfuscate the issue by the way):

$$FI_i = MKT_i \times (P/B_{MKT} / P/B_i)$$

Here FI_i is the weight of the stock in the Fundamental Index, MKT_i is its weight in the traditional market capitalization weighted index, P/B_{MKT} is the price-to-book of the market-cap weighted index, and P/B_i is the price-to-book of firm *i*. Forming a Fundamental Index over multiple measures, something we endorse versus a single measure, complicates the mathematics but doesn't change the spirit one iota. We dub this "the simplest equation in the world" with acknowledgment of the hyperbole in that sentence (Aristotle's A=A, among others, might beat us out for that title).[8] Clearly, the weight of firm *i* in the Fundamental Index is a direct function of relative valuations, as measured by P/B, of firm *i* and the market itself.

Remember, value strategies have been around forever (e.g., Graham and Dodd [1934], Victorian England) and systematic value strategies have been studied in depth since at least the mid-1980s (e.g., Rosenberg, Reid and Lanstein [1985] and Fama and French [1992]) and likely far longer. There is likely no one in investing who would say, if the question was posed this way, that systematically overweighting low P/B stocks and underweighting high P/B stocks versus a capitalization weighted index using a simple formula was anything other than a pure value strategy.

If you like regressions, take the monthly data from Ken French's website mentioned earlier and the backtest of Fundamental Indexing in large capitalization U.S. stocks available on Bloomberg[9] from 1962 through early 2014. The left-hand side subtracts out the returns of the cap-weighted market portfolio from the monthly backtest of Fundamental Indexing such that we measure how much the Fundamental Index beats the cap-weighted market over this period (note, we expect it to win as value has won over this period). The right-hand side is simply Fama and French's famous HML factor, the return spread between a diversified portfolio of low price-to-book and high price-to-book stocks (a factor, to our knowledge, that nobody disputes is pure "value"). Here are the results with *t*-statistics in parentheses:

[8] Arnott, Hsu, and West [2008] actually present this formula even though it originally appeared in Asness [2006]. While they concede it shows that "in any snapshot in time" Fundamental Indexing is "pure value," they go on and try to explain why it is not pure value because it is a different amount of pure value at different times.
[9] "FTSE RAFI US 1000 Total Return Index" with Bloomberg code "FR10XTR Index".

Fundamental Index – Market = -04 bps per annum + 0.37 x HML
(-0.10) (35.2) R-squared = 66%

The Fundamental Index delivers no additional average returns beyond those of value, as measured by Fama and French's HML. The intercept is -4 basis points, meaning after adjusting for "pure" value using Fama and French's HML factor, Fundamental Indexing underperforms by 4 bps (though it is statistically no different from zero, as indicated by the -0.10 *t*-statistic). The enormous 35.2 *t*-statistic on HML testifies to the strong relation between Fundamental Indexing and value. Adding other factors or changing the above factors (e.g., using the version of HML from Asness and Frazzini [2013], adding the famous momentum and size factors, defining value more broadly than HML using multiple measures like FI does, etc.) can change the results, moving the intercept in either direction, but the very tight relation with value remains.[10]

Let us try one for fun and a bit of insight. Asness and Frazzini [2013] find that the Fama-French value factor unnecessarily lags price in its formation. They propose an alternative version of HML that uses up-to-date prices rebalanced monthly.[11] Fundamental Indexing rebalances annually, but unlike Fama and French's HML uses up-to-date prices when it does so (Fama and French's HML uses a six month lag in price when rebalancing). On this dimension, therefore, Fundamental Indexing is somewhere between Fama and French and Asness and Frazzini. What happens when we add the Asness and Frazzini version of HML (called here HML-DEV after that paper's title) to the regression?

Fundamental Index – Market = -06 bps per annum + 0.21 x HML + 0.18 x HML-DEV
(-0.20) (14.7) (14.8)
R-squared = 75%

It seems Fundamental Indexing, as its design would suggest, comes out almost dead in the middle of these two HML constructs (*t*-statistics of approximately 15 on both), with an improved R-squared of 75% from the 66% on just Fama and French's HML alone (remember, this is a

[10] As noted by Arnott, Hsu, and West [2008] this type of regression compares a long-only portfolio to a long-short portfolio, potentially and probably biasing it against the long-only portfolio. That's true, but it is precisely why we subtract the market return from the left hand side of the regression. However, Arnott, Hsu, and West [2008] make similar comparisons when they state: "It's striking to note that the 0.3 percent Fama-French alpha on Table 10.3 *soars* to 1.1 percent in the Fama-French-Carhart analysis. This is 1.1 percent of return, which is *utterly* unexplained in a Fama-French-Carhart model!" Ignoring the hyperbole ("striking", italicizing the already hyperbolic "soars", "utterly", and of course the exclamation point!) and the fact that the intercept is always by definition "utterly unexplained" by the model on the right-hand-side, we think this increase in intercept is real and is happening for precisely the reason documented in Asness and Frazzini [2013]. They are right in that the intercept is positive, but they think and imply that it comes from something that has to do with Fundamental Indexing, not the more mundane improvement found for any value strategy that rebalances with more current prices. Also, note Asness and Frazzini [2013] framed this improvement as a small tweak to a value strategy leading to a slightly better value strategy – not a brand new breakthrough.

[11] The data behind this study is available at https://www.aqr.com/library/data-sets for any who want to experiment themselves.

75% R-squared despite the Fundamental Index being formed on four value measures and HML and HML-DEV just on one). The intercept also drops another 2 bps to -6 bps, but it is still not reliably different from zero. (If we simply regressed on the average of HML and HML-DEV, we get a "modest" *t*-statistic of 43.3, and you thought the earlier 35.0 was impressive?)

Again, specific results will of course vary based on specific regressors. But the core result, that Fundamental Indexing loads gigantically on value, is very robust. We can have reasonable arguments about whether a *t*-statistic of 2.0 or 3.0 should be a standard of significance, but you do not argue about a *t*-statistic of 35 or 43. Now, the R-squared is certainly not 100% (one of our more obvious statements), but it's not 100% because different specific choices were made in constructing the Fundamental Index versus HML. For one of many important differences, Fundamental Indexes use multiple measures of value (which we like, e.g., Frazzini et al. [2013]) while both forms of HML use only price-to-book. Does that make one "value" and the other not? Of course not. Though you wouldn't know it from some of the hyperbole.[12]

To be clear, there is room for many good fairly-priced value strategies. We are believers in the value effect and do not consider it our, or anyone else's, private sand box. Fundamental Indexing does several things we like in a value strategy: it uses multiple measures, it uses up-to-date prices when rebalancing, and it does some implicit timing based on the size of valuation differences across stocks that, despite adding only modest benefit historically, we find intuitively appealing. It is a very clear, simple, even clever way to explain and implement value investing.

Still, it is obvious that it is only a systematic value strategy and a simple one at that. It is not unrelated to value (the story 10+ years ago) or related to but still different from value (the story now). It is exactly value. Proponents of Fundamental Indexing should be absolutely free to argue (quite possibly successfully) why their version of value is better (and be prepared for the counter-arguments).

But the arguments that it is not value and pure value at that should end. It is literally a one line simple value tilt. It might be a very good value tilt. It has in fact proven very popular. Had it been originally marketed as a pure simple value tilt perhaps it would have taken off as successfully as it has, perhaps not. We will never know, but what we do know and should all acknowledge is that it is indeed value. Then we can all get on with arguing in perpetuity over whose value (yes, value!) portfolio is better.

[12] Another way it is claimed that FI is not "value" is it apparently beats the Russell 1000 Value index, which is the straw man here standing in as the only pure "value" strategy. Well, the Russell 1000 Value is an odd beast, as it is not pure price based but considers specific measures of growth, too. We don't know anyone who considers it the only standard of what is "value." And, even if it were the standard of value, beating it only implies you may have a *better* value strategy, not something different from value.

Fact: Profitability, or quality measures, can be used to improve value investing and still be consistent with a risk-based explanation for value.

Some have argued that using profitability to enhance a value strategy is inconsistent with an efficient markets view of the world (and hence may be one reason some choose not to do it), yet we don't believe that is necessarily true. The efficient markets hypothesis (EMH) states that all information should be incorporated into prices, such that any return predictability has to be about risk premia. Nowhere does the EMH state that all firms should have the same price or the same price multiple, such as B/P.

In fact, the use of profitability to enhance value strategies can be consistent with an efficient or inefficient markets view of the world. By cleaning up valuation ratios, that is identifying which firms have low (high) B/P because they are more (less) profitable rather than less (more) risky, profitability helps identify the riskiest (highest expected return) assets from an efficient markets perspective. From an inefficient market scenario, profitability helps find the most underpriced assets, considering that price should vary with quality, with the best hopes for higher future returns. Put simply, under either hypothesis not all firms should have the exact same B/P and measures such as profitability can help remove the variation in B/P that comes with variation in quality and not expected return. Both stories provide a role for profitability making valuation ratios more informative (either about risk or mispricing). Of course, as we often do, we will argue reality is likely best described as a mix of both stories.

Graham and Dodd actually advocated the use of profitability and other quality measures to "clean up" value. That is, while we argued above they were more systematic than Buffett (at least Graham was), they were not "pure" value investors as we use the term here. Below are the main criteria for security selection from The Intelligent Investor (revised in 1973):

1. Adequate size.
2. A sufficiently strong financial condition.
3. Continued dividends for at least the past 20 years.
4. No earnings deficit in the past ten years.
5. Ten-year growth of at least one-third in per-share earnings.
6. Price of stock no more than 1.5 times net asset [book or "balance sheet"] value.
7. Price no more than 15 times average earnings of the past three years.

Only the last two would be considered a valuation metric by most definitions. In our view, the rest would fall under the category of identifying growing high quality companies.[13] In fact, the criteria listed for security selection is very consistent with the Peter Lynch (the famous portfolio manager of Fidelity's flagship and extremely successful Magellan fund) concept of "growth at a reasonable price" (or "GARP").

The idea again being that all stocks should not necessarily sell at the same valuation ratios, which a pure value investment strategy assumes. The fact that pure value investing, ignoring this truism, works so well, despite the illogical implied view that all valuations should sell at the same price, is testament to its power as an investment tool. A very good strategy can survive a little noise.[14] However, one can do even better by recognizing that valuation ratios do not need to be treated exactly the same. Using measures of quality of earnings or profitability can identify the cheap *and* profitable firms that can give an even bigger boost to a portfolio. In this way, persistence in profitability combined with value metrics helps to identify the cheapest, best value opportunities.

Put differently, simple and pure value investing essentially requires buying all cheap companies, many of which can be the really dodgy companies in very poor shape, betting that they will rebound in the future. That bet seems to be a good one on average because the prices these value companies sell for seem low. But, because this strategy systematically buys all cheap companies, it also buys some firms that are cheap for a reason and will continue to underperform. That is, pure value investing makes no distinction between what type of low-priced stocks one should invest in. Adding measures like profitability or quality to the process makes relying on that notion considerably less true. Using value combined with measures of firm quality together can identify the cheap and promising companies.

So, value and growth-like strategies like profitability (and momentum – though we focus on profitability here as we've fought the battle of momentum already!) are not incompatible in theory, but what do the data have to say about their combination? Using publicly available data, Exhibit 1 reports the annualized Sharpe ratios of value, momentum, and profitability (the latter also taken from Kenneth French's website which is a portfolio that is long profitable firms and short unprofitable ones called RMW), as well as various combinations of them. Since profit

[13] Frazzini, Kabiller, and Pedersen [2013], in analyzing Warren Buffett's public track record, found that Buffett indeed bought cheap stocks, as defined by the pure value measures, but he also bought safe (i.e., low risk) and high quality (i.e., profitable, stable, growing, and high payout ratio) stocks. Accounting for these factors helped explain a large part of Buffett's performance.

[14] To be precise, when you sort stocks only on a measure such as price-to-book, and prefer the cheaper ones, you are implicitly saying that firms should all sell for the same price-to-book and any cheaper ones are more attractive. Of course, the question is can you identify the systematic characteristics that should lead one to rationally pay more or less for some firms? We argue profitability and others pass this test. We also argue that the fact that "pure" value is still so effective says that while you can improve by accounting for these quality measures, the cross-sectional differences in expected returns identified by pure value are large enough to survive the sub-optimal act of ignoring them.

measures are only available beginning in July 1963, the results cover the 1963 to 2014 period and in each case use the long-short version of the factor.

Exhibit 1: Combining Value with Momentum and Profitability

	Value	Profitability	Momentum	60/40 Val/Prof	60/40 Val/Mom	33/33/33 Val/Mom/Prof
Sharpe ratio	0.46	0.42	0.57	0.58	0.79	0.84

As the exhibit shows, a simple 60/40 combination of value with profitability improves value's Sharpe ratio from 0.46 to 0.58 over this time period. Further, a 60/40 combination of value with momentum results in an even bigger improvement in Sharpe ratio of 0.79. Importantly, as the last column of Exhibit 1 shows, with a 1/3 equal weighting of value, momentum, and profitability, the improvement in Sharpe ratio is even higher at 0.84.[15] Hence, a value portfolio's Sharpe ratio almost doubles by combining it with growth-like strategies such as momentum and profitability. A simple optimizer will choose positive weights on each to maximize the Sharpe ratio of the portfolio (in fact, the optimized weights are not too far off from 33% in each.)[16]

While adding measures like profitability to improve a value strategy can still be consistent with an efficient markets/risk-based view, that story works best if profitability is both negatively correlated with value and does not carry a positive premium on its own. For example, all-else-equal, being negatively correlated with (hedging) the value factor should imply a negative expected return, so if the return is not as negative as expected, or is zero, then it can add significant diversification benefits. Profitability (and especially momentum) is strongly negatively correlated with value. However, its returns are not simply zero or less negative than they should be, but rather are strongly positive. This, of course, makes profitability an even more valuable factor to add to value, but, at the same time, makes it harder to reconcile from an efficient markets point of view. If profitability is merely "cleaning up" value, something consistent with both risk and behavioral stories, then its efficacy on its own would be flat or neutral. So, given that profitability has a strong positive return premium, it's also doing something more by owning higher quality more expensive stocks. For proponents of a risk-based view of the value premium, this presents a challenge since it is these expensive high

[15] We note again that these findings are exceptionally robust to testing in other countries, and for use in other asset classes where appropriate (momentum always has an analogy for other asset classes, profitability sometimes).

[16] Of course, the returns above are to long-short portfolios and all before trading costs and do not include taxes or other practical costs an investor might face. However, the diversification benefits of combining value with momentum and profitability also extend to trading costs and tax considerations, and to long-only portfolios particularly when viewed versus a neutral benchmark (see Frazzini et al. [2012] and Israel and Moskowitz [2013b]). After taxes and trading costs, the historically optimal portfolio has still been far from pure value, including sizeable weights to momentum and profitability.

quality stocks that also enjoy a positive premium. That fact seems more consistent with behavioral explanations, where high quality firms are mispriced (undervalued), but a harder nut for risk-based theories to crack.[17] Our view is that most, if not all, of these factors work for a combination of reasons – both risk and behavioral – and these results fit nicely into that paradigm.

So, whether you are a systematic, diversified value manager or a concentrated active value manager (like Buffett), adding measures of profitability as another bona fide factor can improve your portfolio greatly, and neither proves or disproves that value is consistent or inconsistent with efficient markets. Clearly, value does not work best alone. Far from it. And, combining it with other intuitive and empirically strong factors such as profitability and momentum builds the best portfolio.

Fiction: Value is "redundant."

Very recently, professors Eugene Fama and Kenneth French advanced a new Five Factor Model (FFM)[18] that adds to their three factor model, consisting of the market (RMRF), size (SMB), and value (HML) factors, two new factors: a "profitability" factor (RMW) and an "investment" factor (CMA). Similarly, Asness, Frazzini, and Pedersen [2014] advance a model that adds a composite "quality" factor that includes within it both profitability and investment related factors with the same sign as in the FFM.

We have already extensively discussed the profitability factor and adding it to standard models seems like a very good idea to us. The second new factor is one related to investment, which is strongly related to payout measures and other factors that many consider as part of a firm's overall "quality."[19]

Some controversy has arisen with regard to the Fama and French new five factor model in that their stalwart value factor, HML, is claimed to be "redundant," in the sense that it adds nothing beyond the other four factors in explaining returns. Enough of a stir was created that Fama and French themselves even decided to write about it, explaining "When we say that *HML* is redundant, what we mean is that its average return is fully captured by its exposures to the other factors of the five-factor model. This means *HML* has no information about average returns that

[17] Perhaps for this reason, some use profitability (and momentum) as a screen rather than as a separate factor in their investment process. We covered this ground in our last paper on momentum, so we will only rehash the highlights here. Basically, the use of other factors as only a screen is a little schizophrenic, or an attempt to be a "little pregnant," because you either believe in these other factors and hence would want to use them effectively, or you do not believe in them and then would not want to use them at all (i.e., you are either pregnant or you are not). Adding only a little bit of a factor via screens is inconsistent with both and only optimal under the most restrictive conditions that are wildly inconsistent with the data. And, even if you only believe in another factor "a little" the proper action would be to give it a little weight in creating your desired portfolio, not to use it as a screen.

[18] See Fama and French [2014].

[19] See Asness et al. [2014].

is not in other factors, so we do not need *HML* to explain average returns."[20] So, should we all stop worrying about, and writing long papers about, say, the facts and fictions related to value investing, instead concentrating on only the other four factors? Should we all stop building investment products with value investing as a core feature and instead switch to these others? We argue no.

In principle there is nothing wrong with value, as proxied by HML, being captured by other sources such as a combination of profitability and investment, implying there are simply better ways to measure and capture the value effect. It would not mean value was useless. It would simply mean its efficacy could be fully captured in other ways. However, we do not believe value is redundant in even this way. And, the reason why has to do with two things we have already discussed along the way: 1) the fact that the Fama-French HML factor, as discussed above, uses price that is highly lagged, and 2), in an ironic twist, the one missing factor that Fama and French have never embraced in their academic work – momentum!

In their new five factor world, Fama and French [2014] explicitly chose to omit momentum, despite the overwhelming evidence that it contributes to explaining returns and is itself not captured by their other five factors (e.g., momentum is not even close to redundant), while simultaneously keeping value, despite their own evidence showing that it is driven out by their other factors. The fact is that the momentum factor they left out resurrects the most famous factor everybody associates with them, value, is not only a bit ironic, but is also very consistent with our long-standing view (emphasized many times in the following papers: Asness et. al [2014], Asness, Moskowitz, and Pedersen [2013], Asness and Frazzini [2013], Asness [2013], and Asness et. al [2015]) that value and momentum are best viewed *together*, as a system, and not stand-alone. Failure to do so can result in erroneous conclusions and poorer investment decisions, of which the apparent "redundancy" of value in the new five factor world is another example.

To be precise, there are two things that resurrect HML within the new Fama and French five factor model that are both related to viewing value and momentum together. The first is including a momentum factor explicitly in the model along with value. The second is to construct a value factor that is separate from momentum by using up-to-date timely, not lagged, measures of prices, a seemingly small change that turns out to have sizeable consequences.

Now, let us walk through the evidence that backs up these claims.[21] For starters in Exhibit 2, we replicate the Fama and French results where HML is made redundant by the other factors. This

[20] See the Fama/French Forum hosted by Dimensional: "Q&A: What does it mean to say HML is redundant?" (http://www.dimensional.com/famafrench/questions-answers/qa-what-does-it-mean-to-say-hml-is-redundant.aspx.)
[21] More details can be found here: https://www.aqr.com/cliffs-perspective/our-model-goes-to-six-and-saves-value-from-redundancy-along-the-way.

is the first panel of Table 6 in Fama and French's [2014] paper. The table reports results from regressing the monthly returns on each of their factors individually from July of 1963 through December of 2013 on the other four factors. In finance geek speak, we are interested in seeing if the two new factors "subsume" or "span" value, which means asking whether a factor can be essentially replicated by a linear combination of the other factors. If a factor is so spanned then it is "redundant."

The first row of Exhibit 2 presents our version (it replicates their version with the sole aesthetic difference being that we present the intercept in percent per annum), reported for HML only.[22]

Exhibit 2: Replicating the Fama and French Five Factor Model (Table 6) and Resurrecting Value

	Intercept	RMRF	SMB	RMW	CMA	UMD	R-squared
HML	-0.48% (-0.46)	0.01 (0.37)	0.02 (0.81)	0.23 (5.38)	1.04 (23.03)		52%
HML	0.52% (0.51)	-0.01 (-0.35)	0.03 (1.04)	0.24 (5.96)	1.03 (23.37)	-0.11 (-5.92)	54%
HML-DEV	0.23% (0.15)	0.06 (2.04)	0.00 (0.11)	-0.02 (-0.30)	0.95 (14.24)		28%
HML-DEV	4.87% (4.74)	-0.01 (-0.32)	0.03 (1.15)	0.07 (1.60)	0.89 (20.01)	-0.52 (-27.31)	68%

Each row reports the regression coefficients, with *t*-statistics below in parentheses. The first column reports the intercept or alpha from the regressions and the last column the R-squared. If the intercept is reliably different from zero, that is statistically significant at a reasonable level of confidence (usually meaning having an absolute value of its *t*-statistic > 2), then the factor contributes to explaining returns above and beyond the other four factors. If the intercept is zero (statistically no different from zero), then the other four factors span or subsume the factor in question. That is, the factor is redundant.

As the first row shows, HML is indeed redundant in this particular model. This is the headliner. The controversial, or at least somewhat jolting, finding.

Digging into the regression results for HML a little deeper, a few things pop out. First, HML is not explained or strongly related to RMRF or SMB, whose coefficients/betas/factor loadings are near zero. However, HML has large and significant exposure to RMW, the profitability factor, and an absolutely gigantic exposure to CMA, the investment factor (with a beta of 1.0 and a *t*-statistic of 23!). This is what eliminates HML's alpha. After accounting for positive covariance

[22] Again, all data save HML-DEV, is from Kenneth French's website.

with profitability (cheaper firms are on average more profitable firms) and conservatism in investing (cheaper firms on average invest more conservatively – this covariance is not as sensitive to other variables) there is no intercept left. In fact, it is even a tad negative.

This means that HML can be reconstructed and is better explained by a combination of RMW and CMA. But, is the reverse true? Can CMA be explained by some combination of HML and RMW? Or, can RMW be explained by a combo of CMA and HML? The answer is no, as separate analysis shows.[23]

What does this mean? Well, it does not mean that value is an ineffective strategy stand-alone, far from it. It simply means that after accounting for the two new factors, value does not add additional returns. If your value-based view of the world has been rocked and you find this at all disturbing, you are overacting. Again, value is still a good strategy. But if you are still breathing heavily we are here to help. The good news: by the end of this myth we will have resurrected value.

The next row of Exhibit 2 adds the momentum factor, UMD, to the regression. Since HML is negatively correlated with UMD (*t*-statistic of -5.92), and UMD is a positive return factor, HML's alpha increases by 1% per annum. That is enough to flip its sign but not enough to get it to statistical significance. So HML is not quite as redundant as before, but not resurrected either.

To save value we are going to have to change it. As discussed in Asness and Frazzini [2013] and mentioned previously, Fama and French's industry-standard construction of HML uses annual rebalancing in June, using book-to-price as the valuation measure to decide "H" and "L", where both book and price are taken as of the prior December. That is, both book and price are six months old upon portfolio formation, and grow to eighteen months old by the time the portfolio is rebalanced next. Regarding the initial six month lag, there is a clear reason to do this for the book value. You do not have accounting information for December 31st fiscal year-end firms on December 31st. The choice of a six month lag is to ensure that an investor would have had this information in real time when forming her portfolio and therefore the backtested results will not suffer from look-ahead bias (i.e., trading on information you would not really have known at that point in time). But, the researcher has a choice with price. You could use, as Fama and French did, price from the same date as book so they match perfectly. That has some intuitive appeal in that the date of price and the date of book are aligned. However, we argue for a mismatch in the timing of book and price for two reasons:

1. If all you knew is price fell dramatically (and vice versa for all these examples) since you last had an accurate matched-in-time book-to-price, your best guess would clearly be that

[23] For more detail, see the post at https://www.aqr.com/cliffs-perspective/our-model-goes-to-six-and-saves-value-from-redundancy-along-the-way.

book-to-price went up because book does not tend to move as much as price.[24] Put more simply, if you are standing in June, and price has sharply fallen since December, you are highly likely to be a cheaper stock and the original Fama-French version of HML misses this additional cheapness. Ignoring this price movement throws away information that is definitely available in a timely way without look-ahead bias. Essentially, irrespective of the correlation with momentum we are about to discuss, or the effect on the "redundancy" regressions, on first principles forming HML with more timely prices turns it into more of a "pure" value factor than is regular HML.

2. A properly constructed value strategy is naturally (and to us quite beautifully) negatively correlated with momentum. If price falls sharply, momentum gets worse, but, as discussed in 1) above, the company usually gets cheaper. Throwing in a six month lag in price that grows to eighteen months before the next rebalance throws away much of this natural, elegant, and intuitive negative correlation.

After making these arguments Asness and Frazzini [2013] construct an alternative to Fama and French's HML by preserving all aspects of their methodology, except allowing the portfolio to rebalance monthly, using last month's price to scale book values. We call this version HML-DEV after the title of that paper "The Devil in HML's Details" and in the third row of Exhibit 2 show the results of re-running the regression by replacing Fama and French's HML with HML-DEV. The result, for the most part, is a non-event. HML-DEV experiences an intercept that is not radically different from using Fama-French HML and still no different from zero. It is now insignificantly positive instead of insignificantly negative. So it is still redundant.

But, now let us add back momentum in the fourth row of Exhibit 2. Resurrection! Finally some meaningfully different results and all from doing two simple things we have advocated for on their own.

Timely value, or HML-DEV, now has an economically and statistically large intercept, even with a very large loading on the positive CMA factor. The negative correlation with the successful momentum factor is just that powerful. Again, value and momentum are best thought of as a system. They are both strong alone, but much stronger together due to their negative correlation (which shows up most clearly when value is defined with timely prices).

Fama and French's latest Five Factor Model is a useful way to summarize the known playing field of factors, and it brings some very good things to the table. However, for reasons we do not find compelling, it leaves out momentum. With no change necessary to the value factor it is absolutely compelling to add momentum back, creating a better Six Factor Model. But, as we have argued for some time and elsewhere, the value factor should in fact be changed to more

[24] See Asness and Frazzini [2013].

timely value. Doing so makes momentum even stronger, and the value factor, rendered distressingly (for those of us who have considered ourselves value investors for many years) redundant by the Five Factor Model, is suddenly and easily resurrected.

Strong proponents of value may rejoice at this news, but at the same time they must face the irony that it was momentum that rescued them.

Fact: Value investing is applicable to far more than just choosing what stocks to own or avoid.

Can value really be applied outside of equities? To many, value is a concept that applies exclusively to stocks, in part because most of the academic literature and evidence has focused on stocks and, relatedly, because the most common way people measure value is by some ratio of accounting-value-to-market-value, such as the book-to-market equity ratio. Since accounting values are non-existent in other asset classes such as bonds, commodities, currencies, etc. value is often thought not to apply to those assets.

But, we can think more broadly about what value investing is trying to accomplish: identify cheap versus expensive assets. If we can measure cheapness and expensiveness in other asset classes, then we can form a value portfolio in other asset classes. There are many reasonable ways to measure value, including reversal of long-term past returns as we alluded to previously. And, since every asset has a measureable return, this is at least one measure of value we can use for any asset class.

In other asset classes, we can form direct fundamental measures of value as well. For instance, in bonds a measure of value is the real bond yield or yield on a bond minus expected inflation. For currencies, deviations from purchasing power parity (PPP) as proxied by the price of a basket of goods in two countries relative to their exchange rate might also indicate cheapness and expensiveness, where in the long-run exchange rates should converge to PPP across countries.

Using some of these value measures, Asness et al. [2013] document significant value return premia in bonds, country equity index futures, commodities, and currencies, as well as for equities globally, from 1972 to 2011. They find not only that there is a reliable value premium in each asset class, but that the correlation of value strategies across asset classes is positive, and more so than passive exposures to the asset classes themselves, indicating that value strategies in different asset classes using different measures, but all tied together by the same theme of identifying cheap versus expensive assets, are still capturing a similar phenomenon. In other words, cheap assets in one asset class move with cheap assets in other asset classes, bonded by an overall value effect that pervades all of these markets.

So, value is more than a narrowly defined equity-only concept and can be applied much more broadly to any asset class. The implications of this are that more robust and diversified value strategies can be created that deliver better and more stable performance. As a case in point,

Asness et al. [2013] show that a diversified value strategy applied across all asset classes more than doubles the Sharpe ratio of a U.S. equity-only value strategy such as HML. Moreover, combining value with momentum, and doing so across all asset classes, improves an overall portfolio by substantially more.

Regardless of whether one takes advantage of these global (across geography and asset class) results, they are out-of-sample tests of the original U.S. results, which makes us more confident that the basic findings, that both value and momentum are effective and more effective together than separately, are robust and not the artifact of data mining.

Fact: Value can be measured in many ways, and is in fact best measured by a composite of many variables.

Simple intuition tells you this has to be true. The opposite idea that a single measure of anything is optimal – given estimation error, data mining concerns, and absent any strong theory – seems at best remote and most likely wrong.

Nevertheless, we put this statement to the test in the data. The predominant, certainly in academia, way to measure value is to use the book value of a firm's equity relative to its market value, referred to as the book-to-market ratio (as we have used in various places throughout this paper). This ratio can also be expressed per share as book-to-price (e.g., dividing numerator and denominator by number of shares outstanding) or if one prefers "high" to mean expensive, the reciprocal is often used as the price-to-book. This particular measure of value has been made popular by Fama and French [1992, 1993, 1996, 2008, and 2012] in a series of papers. But, we know of no theoretical justification for it as the true measure of value versus other reasonable competitors. In fact, Fama and French [1996] use a variety of fundamental-to-price ratios such as earnings-to-price and cash flow-to-price, as well as other measures of value such as dividend yield, sales growth, and even reversal of the past five-year returns (the "poor man's" value measure).

The results are consistent across measures, and the portfolios constructed from different value measures yield highly correlated returns. Exhibit 3 reports summary statistics on HML-style portfolios using different value measures to rank stocks. These portfolios are taken from Kenneth French's website and pertain to the top 30% of stocks (value stocks) minus the bottom 30% of stocks (growth stocks) based on book-to-market equity (BE/ME), earnings-to-price (E/P), cash flow-to-price (CF/P), dividend yield (D/P), and negative past five-year returns.

Exhibit 3: Single vs. Multiple Measures of Value

1951 - 2014	HML Based on:					
	BE/ME	E/P	CF/P	D/P	-5-year return	Composite
Average	3.6%	5.3%	4.5%	1.8%	2.5%	3.5%
Stdev	9.9%	9.8%	9.9%	11.5%	8.2%	8.1%
Sharpe ratio	0.36	0.54	0.45	0.15	0.30	0.43
Correlation to HML (BE/ME)	1.0	0.8	0.8	0.6	0.5	0.9

While there is some variation in returns across the different measures, all of the HML-style portfolios based on different value measures produce positive returns and are highly correlated.

The last column of the exhibit reports an HML-style portfolio using a composite (simple equal-weighted average) of all value measures. As the table indicates, using a composite of value measures results in a more stable portfolio, as indicated by the lowest volatility relative to all of the individual measures. Comparing the traditional HML portfolio based on BE/ME only versus the HML-style portfolio based on the composite of all five value measures, the average returns remain about the same, but the volatility of the composite HML portfolio is 20 percent lower, resulting in a modestly higher Sharpe ratio, even though the correlation between them is 0.9.[25]

It is worth noting that the valuation ratios, such as BE/ME, E/P, and CF/P, deliver better and more robust results than more tenuous measures such as negative past five year returns and dividend yield. This makes sense since past returns do not contain any information about the firm's fundamentals and because many firms (increasingly so, see Fama and French [2001]) do not pay dividends. Hence, we would expect both of these measures to perform worse than the other valuation ratios. A simple composite of just the three other valuation ratios would generate a value strategy that produces an average 4.5 percent return per year with an annualized Sharpe ratio of 0.48.

Thus, not only is book-to-market not the only measure of value, but an average of multiple measures results in a somewhat better and more stable portfolio. This is intuitive. Each individual measure has error to it (due to accounting mis-measurement, missing accounting items for some firms, and random errors), so an average of measures helps reduce noise.

[25] Regressing each measure on the market, size, and momentum factors, again from Ken French's website, yields a similar superiority for the composite with the *t*-statistic of its alpha, or intercept from the regression, being higher than those of all single value measure portfolios. In addition, all of the value portfolios exhibit similarly negative correlations with the momentum factor, highlighting that different value measures also yield similar results in terms of their relation to other factors, such as momentum.

Frazzini et al. [2013] and Israel and Moskowitz [2013a] also show that multiple measures of value produce more stable value portfolios that deliver higher Sharpe ratios, higher information ratios, and more robust returns. As with any systematic process, unless theory dictates preferring one metric to all others, an average of reasonable and sensible measures is generally the best and most robust approach.

There is an additional advantage to using multiple measures as well, which similarly relates to the ability to reduce errors. The out-of-sample performance of a strategy will usually be better (i.e., more closely match the backtest) when using an average of multiple measures. As with any specific sample of data, you will always find some measures that work particularly well in sample and some that do not (e.g., E/P vs. D/P in Exhibit 3). However, without theory telling you a priori why one measure should outperform another, this is largely due to chance. As a consequence, you would not expect that same measure to outperform out-of-sample. Taking an average of multiple measures guards against picking one particular measure over others that happened to work well in one particular sample. In other words, it helps prevent data mining by extracting more of the signal and avoiding over fitting errors.

In Exhibit 4 we report the Sharpe ratios of each value measure separately by decade from 1951 to 2014. The highest Sharpe ratio in each decade is highlighted in grey, while the lowest is highlighted in black. Since we know ex post that E/P produced the highest Sharpe ratio, and D/P produced the lowest, over the full period, E/P should have stronger decade by decade performance than D/P, which it does. However, as the table shows, E/P produced the highest Sharpe ratio in only two decades, the first two (1951-1960 and 1961-1970). Over the last four decades, a different value metric produced the highest Sharpe ratio each time, including D/P from 1981-1990, which is the lowest Sharpe ratio value strategy over the full period. This is just an informal, but we think informative, way to see that attempting to choose the best single measure from theoretically similar ones is both dangerous and at best, over the long-term, not productive.

On the other hand, looking across all measures of value in each decade, we see times when all value measures do better or worse. For instance, 1961 to 1970 was a great time for value no matter how it was measured. And, 1991 to 2001 was not a very good period for value in general. Consequently, the composite index of all five value measures, reported in the last column, is better able to capture the true variation in value's returns by averaging out the errors and idiosyncrasies associated with any singular measure.

Exhibit 4: Sharpe Ratios of Different Value Measures by Decade

	B/M	E/P	CF/P	D/P	-5-year return	Composite
1951-1960	0.28	1.16	0.82	0.08	-0.83	0.51
1961-1970	0.78	1.02	0.86	0.34	0.77	0.85
1971-1980	0.51	0.33	0.54	-0.11	0.37	0.35
1981-1990	0.44	0.29	0.26	0.48	0.04	0.37
1991-2000	0.01	0.39	0.03	-0.15	0.74	0.20
2001-2014	0.28	0.39	0.45	0.30	0.29	0.45
Highest	Lowest					

Bottom line: however you identify value, cheap assets outperform expensive ones. And, no single measure of value is demonstrably better than another. But, an average of multiple measures is typically best.

Fact: Value standalone is surprisingly weak among large cap stocks.

Many academic studies argue and show that return predictability is stronger among smaller stocks, which is true for value, too, and for some potentially good reasons.[26] However, in the case of value, its return predictability as a standalone factor is fairly ineffective among large stocks.

From Kenneth French's data we can see how different large and small cap value is, and how weak the large cap results are. Exhibit 5 looks at "HML small" which goes long cheap and short expensive only among small stocks, "HML large" which does the same only among large caps, and repeats the results for regular HML which as a portfolio is simply the average of the portfolio returns of HML small and HML large. The average market-adjusted returns of these portfolios are reported over four sample periods: 1) the longest period for which Kenneth French provides data on HML (starting in July 1926 and running to July 2014), 2) the out-of-sample period beginning in July 1926 and ending in December 1962, covering the out-of-sample period *before* the start date of many of the original academic studies on value, including Fama and French's seminal papers [1992, 1993] on the 3-factor model, 3) the period from January 1963 to December 1981 largely covering the in-sample period of those academic studies, and 4) the period from January 1982 to July 2014 covering the out-of-sample period *after* the original value studies. We also include *t*-statistics on the significance of the average returns to formally test whether they are reliably different from zero.

[26] Penman, Reggiani, Richardson and Tuna [2013] argue that the accounting system encourages firms to reflect risky activities through deferrals and accruals that will depress current earnings during risky times and create a wedge between earnings and book multiples, where book values help identify risky stocks. The importance of book values increases where earnings growth is higher and more uncertain, as in small stocks.

Exhibit 5: Large and Small Cap Returns of Value

Sample	Value premia (market-adjusted returns)		
	HML small	HML large	HML
1926-2014	5.5	1.7	3.6
(t-stat)	(4.07)	(1.16)	(2.81)
1926-1962*	2.5	0.1	1.3
(t-stat)	(1.14)	(0.06)	(0.63)
1963-1981	6.6	6.0	6.3
(t-stat)	(3.15)	(2.52)	(3.14)
1982-2014*	9.2	1.7	5.5
(t-stat)	(4.78)	(0.93)	(3.15)

*Out-of-sample periods from the original academic studies.

Over the entire sample period, the market-adjusted return to value within small cap stocks is a significant 5.5% per annum, but within large cap it is an insignificant 1.7% per annum (in other words, not reliably different from zero).

Looking at the sub period results, the only period where there seems to be a significantly positive HML premium among large cap stocks is over the in-sample period from 1963-1981, when the bulk of the original academic work on value took place. Over both out-of-sample periods – prior to these studies from 1926 to 1962 and after these studies from 1982 to 2014 – there is no evidence that there exists a healthy value premium among large cap stocks. Even over the entire 88-year sample period that includes the in-sample evidence, HML among large cap stocks does not yield significantly positive returns!

This may come as a surprise to some readers who only remember the original academic studies using data from 1963 to the early 1980s where large cap value does seem to work. However, upon further review and revisiting the data and updating the analysis, there is not, or perhaps there never was, a strong stand-alone value premium among large caps.

Pushing this a bit further also reveals something not generally appreciated about the construction of HML (the benchmark by which most researchers measure value). HML, which is an equal-weighted combination of HML small and HML large, by construction gives much more weight to small than a simple passive cap-weighted value portfolio would, leading to better looking results. The last column of Exhibit 5 shows that HML looks alive and well in every period except 1926 to 1962, precisely the same periods over which small cap HML performed well. While HML is touted and used as a benchmark and often thought of as a passive portfolio, it is actually a monthly rebalanced equal-weighted portfolio of small cap HML and large cap HML, whereby giving 50% weight to small cap HML significantly overweights exposure to small cap value relative to a cap weighted value benchmark. Furthermore, since the risk of small cap stocks

is higher than large caps, the exposure to small caps is even greater and more imbalanced from a risk perspective. A purely cap-weighted value portfolio with market-cap weights would look much like HML large, as the large stocks would dominate in cap-weighting which does not reveal a significant return premium over the full sample period (it would likely look slightly better as it would have very small, but positive, exposure to smaller stocks).

For a more detailed study on this topic, see Israel and Moskowitz [2013a], who find that the value premium is virtually non-existent among large stocks.

Despite the weakness of the large cap results, we are still big proponents of value investing even among large cap stocks. Why? Because the weak stand-alone evidence of large cap value should not be confused with value's very valuable contribution to a portfolio, particularly one with momentum or profitability in it as we showed earlier.

Exhibit 6 looks at small and large cap value separately in combination with momentum. Even though small cap value is a higher Sharpe ratio strategy than large cap value, it is still greatly improved by combining it with momentum – raising the Sharpe ratio from 0.48 to 0.82. But, for large cap value, which by itself only generates a 0.25 Sharpe ratio, combining it with large cap momentum produces a robust 0.65 Sharpe, which is not so far off the combination of value and momentum for small caps. In other words, combining value with momentum and viewing them as a system, the results for small and large cap stocks are similar, and more importantly are quite strong for large cap. Once again momentum rides to value's rescue!

Exhibit 6: Sharpe Ratios of Value and Momentum Combinations in Small and Large Caps

Small Cap			Large Cap		
		60/40			60/40
HML	UMD	HML/UMD	HML	UMD	HML/UMD
0.48	0.49	0.82	0.25	0.49	0.65

While large cap value as a stand-alone strategy by itself is rather weak, combining it with momentum is a powerful combination that results in a very attractive portfolio.[27]

Fiction: Value's efficacy is the result of a risk premium not a behavioral anomaly and is therefore in no danger of ebbing going forward.

There are two parts here. One is that value is a risk premium, meaning a value strategy delivers attractive long-term returns by taking a compensated risk in a rational market. While we certainly would not argue against this being part of the story, we will argue that the evidence is

[27] In fact, resurrecting large cap value is quite easy. Merely adding 20% of momentum to large cap value bumps the Sharpe ratio up from 0.25 to 0.40.

far from conclusive. The academic community continues to debate this notion, and our best guess is that both risk and behavioral causes are at work. Second, even if value is a risk premium that does not mean it could not disappear in the future. Conversely, behavioral anomalies do not have to disappear. Thus, the first part of the statement does not necessarily follow regardless of whether value is a risk premium or a behavioral anomaly.

The premise of the statement is that value represents compensation for risk in an efficient market and that *because* of this you can expect it will continue to deliver the same risk premium in the future. There is a more subtle implication: a return premium that is not tied to risk or possibly reflecting an inefficiency in markets, should be expected to disappear as investors arbitrage it away. Neither are necessarily true.

First, it is important to acknowledge that there is a lively and healthy debate regarding the economic explanation behind value. No model of value is so compelling that a consensus exists of its explanation. Risk-based stories center on the value premium being compensation for bearing some type of systematic risk and going through periods, often prolonged, of underperformance. Value suffering from 1998 to 2000 during the technology run-up, the great depression and the global financial crisis, could be supportive of the risk-based story, especially the last two examples as these were particularly painful times to suffer.[28] Fama and French have suggested that distress risk may be related to value's risk premium, where value stocks have a higher beta on some market-wide distress indicator. Evidence for this theory is somewhat mixed as pointed out by Campbell et al. [2011]. Furthermore, Novy-Marx [2012] argues that the results on firm profitability and its interaction with value provide a real challenge to the distress risk story. We have hit on this earlier. If something (value) is purely a distress premium it would be odd that you can mitigate that distress (buying stronger more profitable companies) and *get paid* to do so. Typically, you have to pay to alleviate risk, not get paid, so, if profitability was a negative return that hedged value and thus was useful it would be a more consistent story. In addition, the notion of distress is hard to reconcile with the evidence on value effects in commodities, for example, where it is difficult to think of what "distress" means.

The behavioral theories focus on investor mis-reaction to information that causes temporary mispricing. For value, a leading story, originally suggested by DeBondt and Thaler [1985] and Lakonishok et al. [1994] and later formalized by Daniel et al. [1998], is that the value premium is driven by investor overreaction. The idea is that value stocks are neglected stocks that investors have fled from and now shun, while growth stocks are glamour stocks that investors have irrationally stampeded toward, causing value stocks to be underpriced and growth stocks to be overpriced. This story, while formulated before it, is often seen to be buoyed by the technology

[28] Those selling value strategies as a risk-based factor should emphasize these bad periods. Put differently, it would be a heroic marketing technique to call something "risk" while presenting no evidence of such risk. That would be like telling your investors they get to earn the insurance premium but will never have to pay. That is a fairy tale. If one says something is entirely risk-based one should also explain why it is sometimes excruciating, even life threatening!

boom and bust of the late 1990's and early 2000's and the corresponding bust and boom of value strategies.

Academics from the rational-risk-based-efficient markets camp continue to wage war with academics from the behavioral-irrational-inefficient markets group over what drives the value premium. While the jury is still out on which of these explanations better fit the data (indeed, the 2013 Nobel Prize committee split the prize between the two camps),[29] almost all agree that the data are undeniable – value offers a robust return premium highly unlikely to be the random result of data mining. And, in our view, both theories have some truth to them as is likely the case with other premia as well, such as momentum and profitability. The world is rarely so bright-lined that one theory is correct and the other completely wrong. Elements of both risk and behavior are likely present in the value premium.

Furthermore, to make things more interesting, or more complicated depending on your perspective, if both explanations have important elements of truth and make up part of the story, nothing says that mix is constant through time. For instance, inefficiency-based behavioral type reasons may have driven much of the 1999-2000 period where value suffered at historic levels, and then soared at historic levels, but that could be the exception that proves the rule. The important point, however, is that both theories offer good reasons to expect the value premium to persist in the future.

If value is a risk premium, however, would that imply that we do not expect it to disappear (and its subtext: if not risk, then it would disappear)? Just because something is related to risk (and we reiterate that for value this is far from certain) does not guarantee it will not get greatly reduced. If risks change or the taste or compensation for risk changes, then so, too, will the expected returns to that risk. Critics of efficient markets have long mistaken time-varying risk premium for a refutation of the idea of efficient markets based risk premia. As long as risks and tastes for risks do not change, then and only then will the premium remain stable and long-lived. It might be perfectly reasonable to believe that these risks will not change, or will change slowly, but then one should state that is the assumption they are making. The claim "it will not change, diminish greatly or, in the extreme, disappear" just because "it is risk" is false.

Conversely, if value is not risk, but due to mispricing, does it follow that the returns to value would necessarily and eventually disappear? No. For mispricing to disappear, either investor biases would have to disappear, or enough capital willing to take the other side would have to dominate trading. Efficient markets proponents often claim the latter will naturally happen, but there are impediments like the limits of arbitrage and the more general arguments in Fama and French [2007] that prevent it from happening, and there is no reason to think that there will not be enough maintained behavior on the part of irrational investors to keep the mispricing going. (In fact, the mispricing can get worse!) As long as the biases, behaviors and limits to arbitrage

[29] See Asness and Liew [2014] for a thorough discussion of these issues from both academic and practitioner perspectives.

remain relatively stable, the premium will be as well. During the 1999-2000 technology episode,[30] while most either bought into tech prices hook, line and sinker or thought them irrational, some thought we were witnessing the rational arbitraging away of the formally effective value strategy. Of course, rarely has an observation been so backwards; value was not being arbitraged away, it was being ignored! And this episode happened many years after value was "discovered" academically and eons after Graham and Dodd, with more and better information available to all in the market to boot.

To be clear, we are not saying that the risk-based stories and the behavioral stories are necessarily on equal footing. Some may reasonably view a risk-based story as less likely to go away than a mispricing behavioral story. Our only point is that *both* risk-based and behavioral stories provide plausible reasons to expect a continued value premium in the future. We are not opponents of, or cheer-leaders for, risk-based or behavioral stories for value. (In fact, we think both likely contribute.) The important point is that both theories give us a good reason to believe in the value premium. Evidence from more than a century of data in plenty of out-of-sample periods, in dozens of financial markets and different asset classes, and with no signs of getting weaker despite investor knowledge of value investing going back at least three decades (see Israel and Moskowitz [2013a] who show no degradation in value's returns), is further testament that the value premium is not likely to disappear soon.

Finally, what if value did disappear?[31] Suppose, despite all of the theory and evidence to the contrary, value had a *zero* expected return going forward. Would it still be a valuable investment tool? The answer is clearly yes, under the condition where the other major risk premia or anomalies remain viable. The reason is because of value's tremendous diversification benefits when combined with other factors, such momentum or profitability. Warning: the following is almost an exact repeat of the analysis in Asness et al. [2014], but this time applied to value instead of momentum. As such, the conclusion is identical so we run the risk of beating a dead horse.[32]

Using Kenneth French's data, we run simple optimizations that maximize the Sharpe ratio of a portfolio combining the market (RMRF), size (SMB), value (HML) and momentum (UMD). Exhibit 7 shows the optimal weight of value as a function of value returns, while holding constant the expected returns of the other factors and the correlations between factors at their long-term averages (1927-2014). Using the average value premium observed in the full sample, the optimization would place about 33% of a portfolio in HML. Moreover, the exhibit shows that even in the extreme case where we assume a *zero* return for value, the optimal portfolio still places a significant positive weight on value (about 13%). The logic is simple. Since momentum

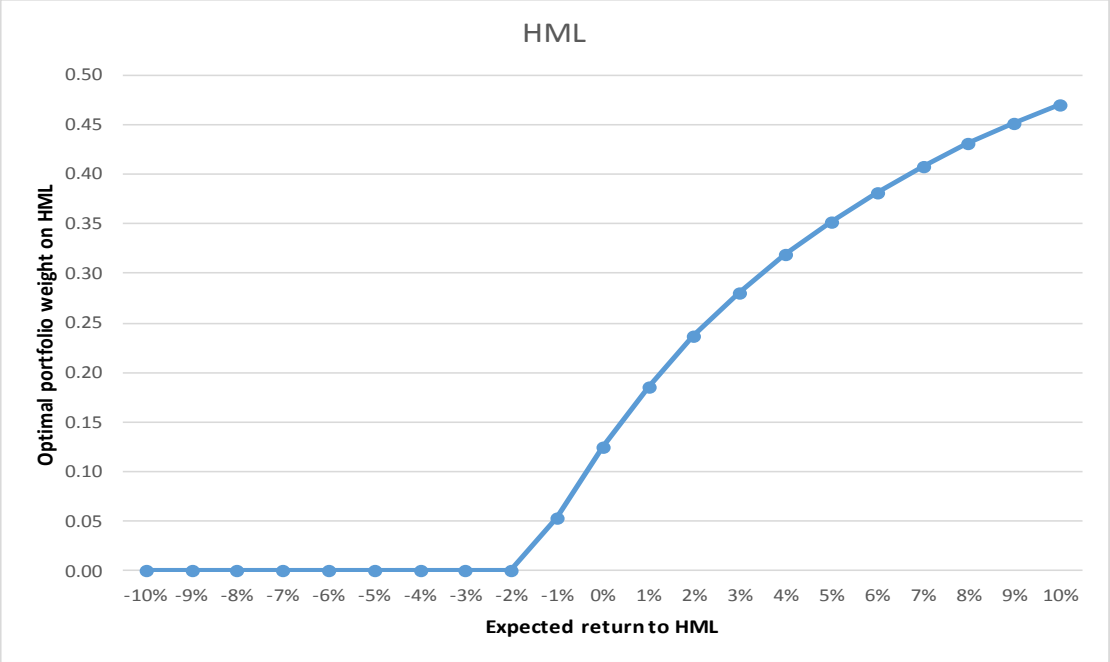
[30] Some of the paper's authors are old enough to have been systematic value investors during this period!

[31] By the way, if the returns to value did disappear over time, a value investor would benefit as the returns converged to zero. Unless you believe the returns have already gone away (which, of course, we do not) then you should not be concerned about this long-term possibility. In fact, you might welcome it.

[32] Not that we condone beating live horses, either. In fact, we would argue it is far worse (if less downright odd).

is a good strategy and value is significantly negatively correlated with it, you would expect value to lose money, and the fact that it breaks even makes it a valuable hedge. The diversification benefits are so great that even a zero expected return would be valuable to your portfolio. Of course, this assumes that the value premium goes away but the momentum premium remains. But, even with this assumption, we think this is strong testimony to the power of diversification across negatively correlated strategies.

Exhibit 7: Optimal Weight Frontier for Value



Another point that is often missed is that not every investor can (or should if it's a risk) hold value (or tilt towards value). Remember, all investment has to, by definition, aggregate to the market. So, for every value investor there has to be a growth investor willing to take the other side. If not, and everyone decided to tilt towards value stocks and away from growth stocks, then the value premium would cease to exist.

For both explanations of the value premium, therefore, a key component is "who is on the other side?" In the case of the behavioral stories, it is clear: those who suffer from behavioral biases or like to chase the glamorous growth stocks and neglect the fallen value stocks. As long as enough of them continue to survive, the value premium remains intact. For the risk-based explanation, the answer is investors who do not want to bear the risk associated with value, or more accurately, those who would pay not to bear it. That is, investors who are long value are provided a risk premium, like an insurance premium, by those willing to pay for this insurance (who are naturally short or underweight value). Of course, again, to fully understand this insurance based explanation we need to fully understand what catastrophe some investors are rationally insuring against.

This then begs the question: if a manager offering value to its clients believes it is related to risk, then why not also offer a growth fund for those wishing to bear less of this risk? In other words, if HML provides a risk premium to investors willing to bear value's risk, should they not also offer LMH to the other set of investors who are willing to pay to eliminate this risk? To our knowledge, no one does (at least explicitly, as some may offer it, losing money over the long term, by accident!).

In summary, the jury is still out as to whether the value premium exists because of risk or behavioral based explanations, and we believe the truth is likely a combination of the two. But whether risk or behavioral, both theories give a plausible reason to expect a value premium to continue to persist..

So, that is it. Those are the facts and fictions of value investing.[33] If you made it this far . . . Wow!

Conclusion

Despite being around forever and formally studied for at least thirty years there is still a lot of confusion surrounding value investing. Now that you have seen the evidence, know where to find it, and can replicate it yourself, hopefully the truth behind value investing will be clearer.

As we stated before, if one wants to challenge the evidence, that is fine, too. As always, if someone discovers something challenging or enlightening versus what we have shown, we welcome it and wish to understand it.

Bottom line: in dispelling the myths about momentum in our prior paper, and detailing the facts and fictions about value in this one, we end up even stronger believers in both factors, and in particular their efficacy when used together.

[33] In case you are wondering what did not make the list, there are two honorable mentions: 1) value is better for a taxable investor than other factors (namely, momentum) and 2) value has a lot more evidence behind it than other factors. Both of these are fiction and covered extensively in Asness et al. [2014] coming from the perspective of momentum investing.

Appendix

Factor Definitions

Below are definitions of the factors used extensively in the paper. For all factors, Kenneth French's data library provides returns of the long and short sides separately, for both large and small capitalization securities separately, all of which we use in this article.[34]

- **RMRF**: represents the equity market risk premium, or aggregate equity return minus the risk free (U.S. Treasury bill) rate. It is the return from simply being long equities at market capitalization weights and, unlike the other factors, is not a "spread" return between one set of stocks and another but between all stocks and cash;
- **SMB** (or "small minus big"): represents a portfolio that is long small stocks and short big stocks to capture the "size" effect;
- **HML** (or "high minus low"): represents a portfolio that is long high book-to-price stocks and short low book-to-price stocks representing "value" investing;
- **UMD** (or "up minus down"): represents a portfolio that is long stocks that have high relative past one-year returns and short stocks that have low relative past one-year returns
- **RMW** (or "robust minus weak"): represents a portfolio that is long stocks that are high on operating profitability and short low ones to capture the "profitability" effect;
- **CMA** (or "conservative minus aggressive"): represents a portfolio that is long stocks that are low on investment and short high ones to capture the "investment" effect.

[34] A link to Kenneth French's data library can be found here if you want to use the data, update the series or check the analysis yourself: http://mba.tuck.dartmouth.edu/pages/faculty/ken.french/data_library.html.

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